Abstract

This thesis is a recollection of several contributions to Number Theory, which I have obtained during the years of my PhD studies. It is divided into two parts, which are essentially independent from each other.

In the first part, we prove a number of results concerning the arithmetic properties of terms of linear recurrences. Given two linear recurrences F and G over a number field $\mathbb{K}$ satisfying some mild hypotheses, we give an upper bound for the counting function of the set $\mathcal{N}$ of positive integers n such that the ratio $F(n)/G(n)$ belongs to a finitely generated subring of $\mathbb{K}$. This makes quantitative a result of Corvaja and Zannier, which states that $\mathcal{N}$ has zero natural density. We investigate also the set of positive integers n which are relatively prime with the nth term of a linear recurrence over the integers, and the set of positive integers n such that the G.C.D. of n and the nth Fibonacci number is equal to a prescribed integer.

The second part of the thesis, on the other hand, consists of two chapters dealing with unrelated topics. In the first chapter, we prove that any sequence $f(n)_{n \geq 0}$, where $f \in \mathbb{Z}[X]$ is a quadratic or cubic polynomial, satisfies a coprimality condition known as *Pillai property*. This extends a result of Evans, who considered the case of f being a linear polynomial, and settles a conjecture of Harrington and Jones. In the second chapter, we study the set of positive integers n which are relatively prime with the nth central binomial coefficient $\binom{2n}{n}$, and we improve a result of Pomerance regarding the upper density of such set.